Name: ______________________________ **Teacher:** **AF SE RC SW MA**

NESA Number: ______________________________

ASCHAM SCHOOL

MATHEMATICS ADVANCED TRIAL EXAMINATION 2024

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using blue or black non-erasable pen
- Calculators approved by NESA may be used
- A reference sheet is provided

Section I **10 marks**

- Answer Questions 1 – 10 on the multiple-choice sheet at the back of this exam paper
- Allow about 15 minutes for this section

Section II **90 marks**

- Attempt Questions 11 – 36.
- Allow about 2 hours 45 minutes for this section
- For questions in Section II, show relevant mathematical reasoning and/or calculations
- Additional writing space is available on pages 33 to 41. If you use this space, please start a new sheet of paper for each question, label it with your NESA number and teacher, and clearly note which question you are answering.

Note: You must submit all components of this exam including all pages, the multiple choice answer sheet, the reference sheet and any extra pages or booklets.

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet at the back of this exam paper for Questions 1 – 10

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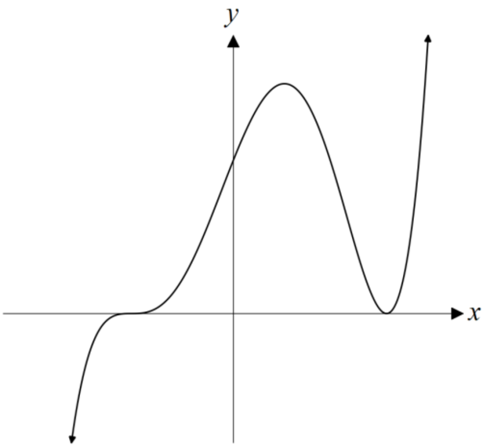
1. An amount of $6000 is invested at an interest rate of 6.5% p.a. compounded annually. How much interest, to the nearest dollar, has been earned after 15 years?

A. $2 333 B. $5 850 C. $9 431 D. $15 431

2. Which of the following could be the graph of $y = (x+3)^2(2-x)^3$?

A.



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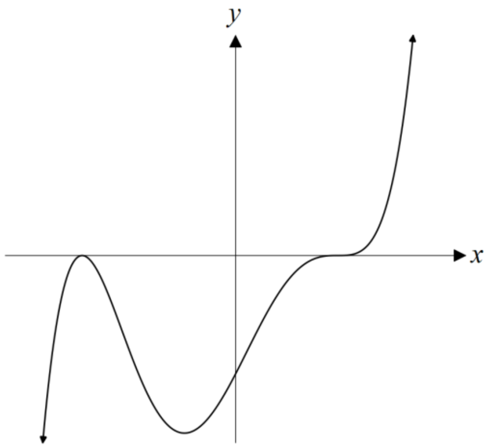
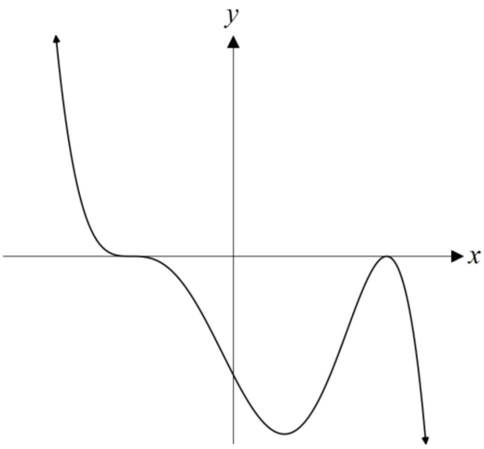
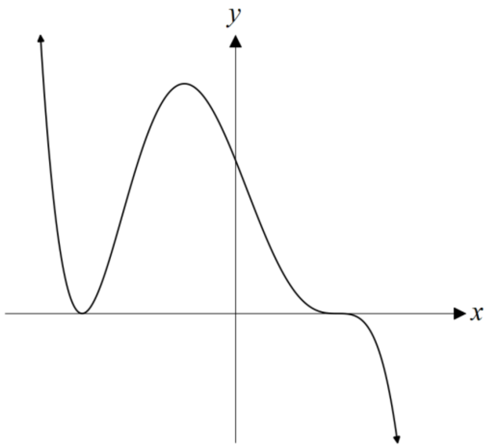


C.



D.



3. The table below shows the present value interest factors for an annuity of $1 invested at the end of each period for various interest rates r and number of periods n.

Table of present value interest factors					
r / n	Interest rate per period (as a decimal)				
	0.0075	0.0080	0.0085	0.0090	0.0095
71	54.89293	54.00754	53.14226	52.29657	51.46995
72	55.47685	54.57097	53.68593	52.82118	51.97618
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74	56.63169	55.68446	54.75957	53.85641	52.97438

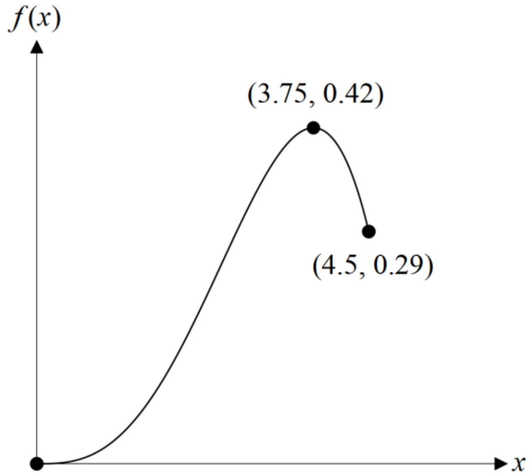
Liv is planning to invest $100 at the end of each month at an interest rate of 0.0085 per month for 6 years. What single amount could Annabelle invest now at the same interest rate, in order to achieve the same future value as Liv in 6 years time?

A. $3 914.42 B. $5 368.59 C. $7 200 D. $13 243.34

4. Which expression shows $\dfrac{45^{2-k}}{6^k}$ as a product of its prime factors in simplified form?

A. $\dfrac{15^{6-4k}}{2^k}$ B. $\dfrac{15^{6-2k}}{2^k}$ C. $\dfrac{3^{4-3k} \times 5^{2-k}}{2^k}$ D. $\dfrac{3^{4-k} \times 5^{2-k}}{2^k}$

5. The probability distribution function of a continuous random variable is shown below.



Which of the following is the mode of this probability distribution function?

A. 0.29 B. 0.42 C. 3.75 D. 4.5

6. Which of the following formulas could be used to calculate the sum of the first n positive **even** numbers?

A. $S_n = n(1 + n)$

B. $S_n = 2(2^n - 1)$

C. $S_n = 2n(n + 1)$

D. $S_n = \frac{n}{2}(6 - 2n)$

7. A random variable is normally distributed with a mean of 0 and a standard deviation of 1. The table below gives the probability that this random variable lies below z for some positive values of z.

z	1.30	1.31	1.32	1.33	1.34	1.35	1.36	1.37	1.38	1.39
Probability	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177

What is $P(z \leq -1.38)$?

A. –0.9162

B. 0.0838

C. 0.4851

D. 0.9162

8. The following transformations are applied to the function $f(x) = x^2 - 3$:

- A vertical dilation with scale factor 3.
- Then a horizontal dilation with scale factor 2.
- Then a vertical translation down 1 unit.

Which of the following is the equation of the transformed function $g(x)$?

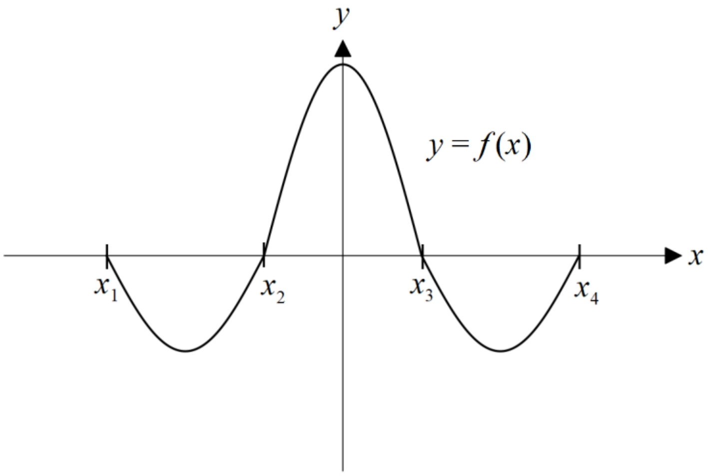
A. $g(x) = 12x^2 - 4$

B. $g(x) = \frac{3}{4}x^2 - 4$

C. $g(x) = 12x^2 - 10$

D. $g(x) = \frac{3}{4}x^2 - 10$

9. The even function $y = f(x)$ is shown below.



It is given that $\int_{x_2}^{x_3} f(x)\,dx = 2$ and $\int_{0}^{x_4} f(x)\,dx = -1$. What is the value of $\int_{x_2}^{x_4} f(x)\,dx$?

A. -1

B. 0

C. 1

D. 2

10. For which values of k is the parabola $y = kx^2 - 2x + 3k + 2$ always positive?

A. $k > 0$

B. $k > \frac{1}{3}$

C. $-1 < k < \frac{1}{3}$

D. $k < -1$ or $k > \frac{1}{3}$

End of Multiple Choice.

Question 11 (2 marks)

A parabola with vertex (2, –5) has a y-intercept of 11. Find the equation of the parabola. **2**

Question 12 (2 marks)

If $3 + \frac{1}{x} = \sqrt{15}$, find the exact value of x. Express your answer with a rational denominator. **2**

Question 13 (3 marks)

A discrete random variable X has probability distribution as shown in the table below, where a and b are real numbers and $E(X) = 2.2$.

x	1	2	3	4	5
$P(X = x)$	0.15	$3a + b$	0	$a - b$	0.05

(a) Find the values of a and b. **2**

(b) Find $\mathrm{Var}(X)$. **1**

Question 14 (2 marks)

Differentiate $y = \ln\left(\dfrac{x^2}{2x+1}\right)$. **2**

Question 15 (2 marks)

Find $\int x^2(1-x^3)^5\,dx$. **2**

Question 16 (2 marks)

Use 3 applications of the trapezoidal rule to estimate $\int_0^{12} \sqrt{x}\,dx$. Give your answer correct to 1 decimal place. **2**

Question 17 (6 marks)

When Alexia plugs her phone in to charge, the battery percentage displayed on her phone is given by the equation $B = -50e^{-0.6t} + 92$. B is the battery percentage and t is the number of hours the phone has been charging for.

(a) Sketch the graph of B for $t \geq 0$. **3**

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(b) What is the maximum charge capacity of Alexia's phone battery? **1**

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(c) At what rate is Alexia's phone charging after being plugged in for 2 hours? Give your answer rounded to the nearest whole. **2**

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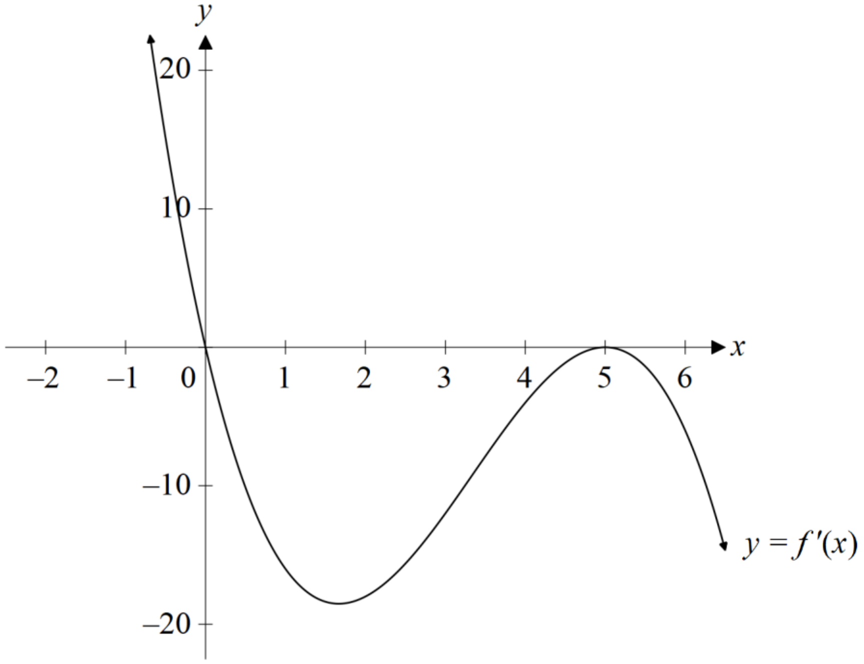
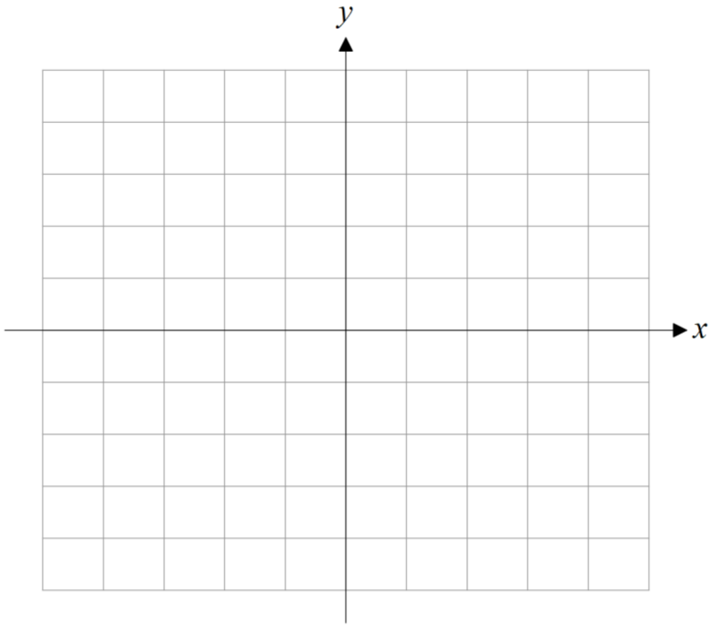
Question 18 (2 marks)

The graph of $y = f'(x)$ is shown below. Sketch a graph of $y = f(x)$ on the same axes, given that $f(0) = 20$. **2**



Question 19 (3 marks)

(a) On the axes below, graph the function $y = |2x + 1| - 3$ labelling all important features. **2**



(b) Hence, or otherwise, solve $|2x + 1| - 3 \leq 0$. **1**

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Question 20 (2 marks)

Find $\int \sin\left(\dfrac{x}{2}\right)\,dx$. **2**

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Question 21 (3 marks)

Amalie is bouncing on a trampoline. Her first bounce takes her to a height of 30 cm above the trampoline. She then lands on the trampoline and continues to bounce. Each time Amalie bounces, she rises to 70% of her previous height above the trampoline. Find the total distance Amalie will cover eventually. **3**

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Question 22 (3 marks)

A function $y = f(x)$ undergoes the following transformations:

- A reflection in the x axis
- Then a horizontal dilation with scale factor $\frac{1}{3}$
- Then a horizontal translation 7 units left

The point $P(x, y)$ lies on $y = f(x)$. The image of P on the transformed function is $P'(-2, 1)$. Find the coordinates of P. **3**

Question 23 (3 marks)

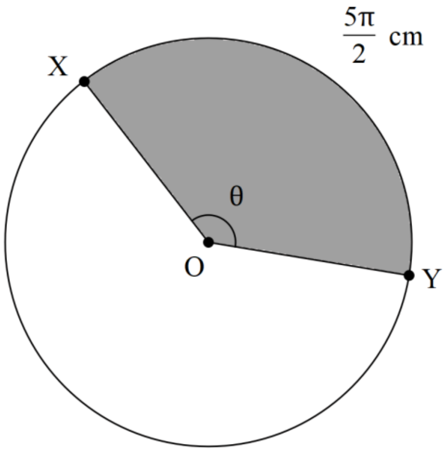
Consider the functions $f(x) = x^2 - 6x$ and $g(x) = \sqrt{x+5}$.

(a) Find an equation for the composite function $g(f(x))$. **1**

(b) Hence find the domain of $g(f(x))$. **2**

Question 24 (3 marks)

In the diagram below, the length of minor arc XY is $\frac{5\pi}{2}$ cm, θ is the angle subtended by that arc, and the area of shaded sector XOY is $\frac{15\pi}{4}$ cm^2. Find the exact value of θ in radians. **3**



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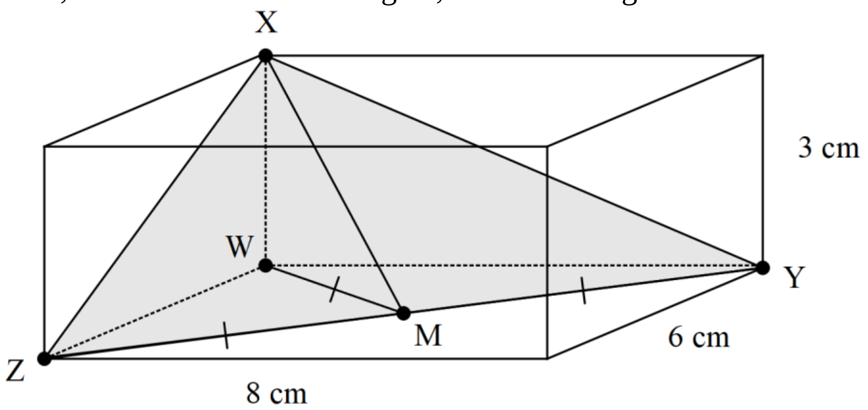
Question 25 (2 marks)

The diagram below shows plane *XYZ* inside a rectangular prism. *M* is the midpoint of the base diagonal *YZ*. Find, correct to the nearest degree, the size of angle *WMX*. **2**



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Question 26 (2 marks)

Find the gradient of the tangent to the curve $y = \dfrac{\sqrt{x-1}}{x^2}$ at the point where $x = 2$. **2**

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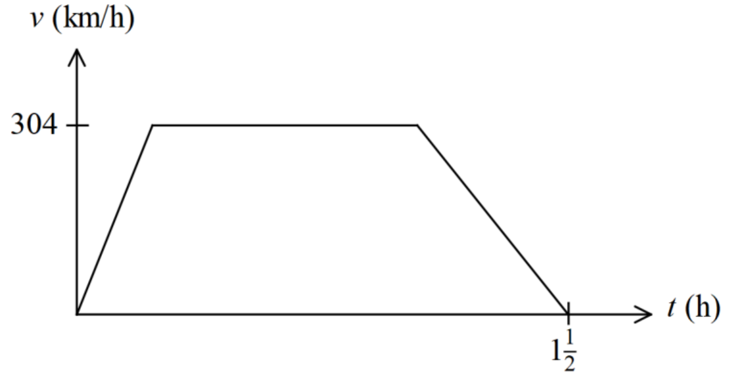
Question 27 (2 marks)

A bullet train is travelling from Sendai to Tokyo. The train leaves Sendai and accelerates at a constant rate until it reaches 304 km/h. It continues at this speed for 45 minutes before decelerating at a constant rate and coming to a stop in Tokyo. The total travel time is $1\frac{1}{2}$ hours. By considering the velocity-time graph shown below, find the total distance the train travelled.

2



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Question 28 (5 marks)

Tish runs a business selling packets of stickers. The number of stickers per packet is normally distributed with a mean of 200 and a standard deviation of 2. In order to balance customer satisfaction and profits, Tish only sells packets that contain between 198 and 204 stickers.

(a) If Tish produces 400 packets of stickers per month, how many of these can she sell? **3**

(b) Given that a packet is unable to be sold, what is the probability that it contains fewer stickers than average. Give your answer to the nearest percent. **2**

Question 29 (4 marks)

May has a mango orchard with 20 trees that each produce 400 mangoes per year. For every additional tree that May plants in the orchard, the output of every tree decreases by 10 mangoes per year.

(a) Show that the number of mangoes, M, produced by May's orchard each year is given by $M = -10x^2 + 200x + 8000$ where x is the number of additional trees planted. **2**

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(b) Hence find the number of additional trees May should plant in order to maximise the number of mangoes grown each year. **2**

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Question 30 (5 marks)

Let $f(x) = \cos\left(3x + \dfrac{\pi}{2}\right) - 1$.

(a) Sketch $y = f(x)$ in the domain $0 \leq x \leq \pi$, labelling all important features. **3**

(b) Hence, or otherwise, solve $\cos\left(3x + \dfrac{\pi}{2}\right) = -1$ for $0 \leq x \leq \pi$. **2**

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Question 31 (8 marks)

At the beginning of January 2023, Hannah decides that she wants to go travelling after finishing Year 12. To save for her trip, she starts investing \$100 at the end of each month into a bank account which earns interest at a rate of 6% p.a. compounded monthly.

The table below shows the future value interest factors for an annuity of \$1 invested at the end of each period.

Periods	Interest rate per period							
n	0.25%	0.5%	1.00%	2.00%	3.00%	4.00%	5.00%	6.00%
6	6.0376	6.0755	6.1520	6.3081	6.4684	6.6330	6.8019	6.9753
7	7.0527	7.1059	7.2135	7.4343	7.6625	7.8983	8.1420	8.3938
8	8.0704	8.1414	8.2857	8.5830	8.8923	9.2142	9.5491	9.8975
9	9.0905	9.1821	9.3685	9.7546	10.1591	10.5828	11.0266	11.4913
10	10.1133	10.2280	10.4622	10.9497	11.4639	12.0061	12.5779	13.1808
11	11.1385	11.2792	11.5668	12.1687	12.8078	13.4864	14.2068	14.9716
12	12.1664	12.3356	12.6825	13.4121	14.1920	15.0258	15.9171	16.8699

(a) Use the future value table above to find Hannah's account balance at the end of December 2023. **1**

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(b) At the beginning of January 2024, Hannah starts working at a new job and decides to increase her monthly investments to \$200. If Hannah continues this for the year, show that her account balance at the end of December 2024 is \$3776.76. **2**

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Question 31 continues on page 24

Question 31 (continued)

(c) At the beginning of January 2025, Hannah starts her trip and stops contributing to the account. Instead, she withdraws $400 from the account **at the start** of each month to cover expenses. The account continues to earn interest at 6% p.a. compounded monthly. Show that Hannah's account balance, A_n, at the end of the nth month of her trip is given by:

$$A_n = 3776.76(1.005)^n - 80400(1.005^n - 1)$$

3

Question 31 continues on page 25

Question 31 (continued)

(d) How many $400 withdrawals can Hannah make from her travel fund? **2**

End of Question 31

Question 32 (4 marks)

The time in minutes taken for Claudia to score her first point in a tennis match can be represented by a continuous random variable X with probability density function given by:

$$f(x) = \begin{cases} ke^x & \text{for } 0 \leq x \leq 4 \\ ke & \text{for } 4 < x \leq 10 \\ 0 & \text{for all other } x \end{cases}$$

(a) Find the exact value of the constant k. **2**

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(b) The median time taken for Claudia to score her first point is less than 4 minutes. Find this median time, giving your answer correct to the nearest second. **2**

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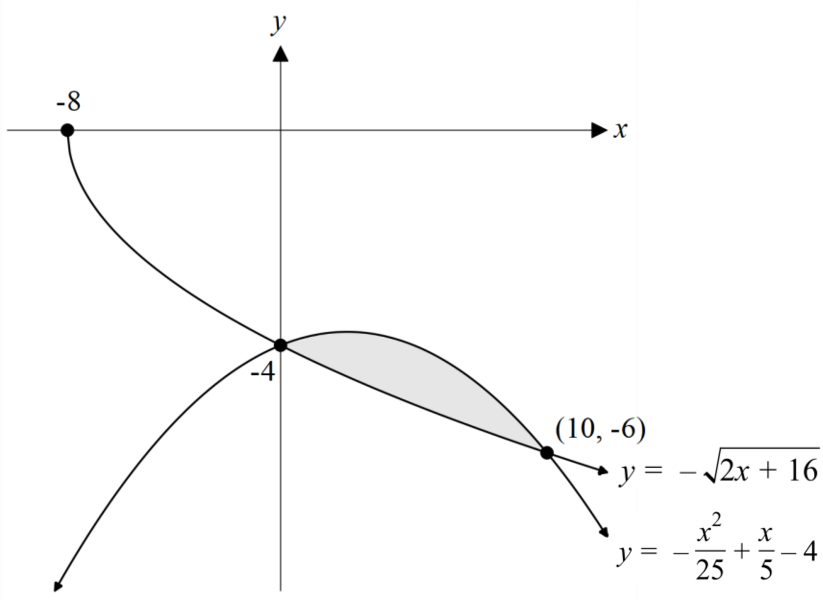
Question 33 (4 marks)

The diagram below shows the graphs of $y = -\sqrt{2x+16}$ and $y = -\dfrac{x^2}{25} + \dfrac{x}{5} - 4$. Find the area of the shaded region. **4**



Question 34 (8 marks)

Let $f(x) = 10e^{-x}(1 - x)$.

(a) Find the exact coordinates of any stationary points on $y = f(x)$ and determine their nature. **3**

Question 34 continues on page 30

Question 34 (continued)

(b) Find the exact coordinates of any points of inflection on $y = f(x)$. **2**

(c) Hence sketch the graph of $y = f(x)$ for $x \geq 0$, showing all important features. **3**

End of Question 34

Question 35 (4 marks)

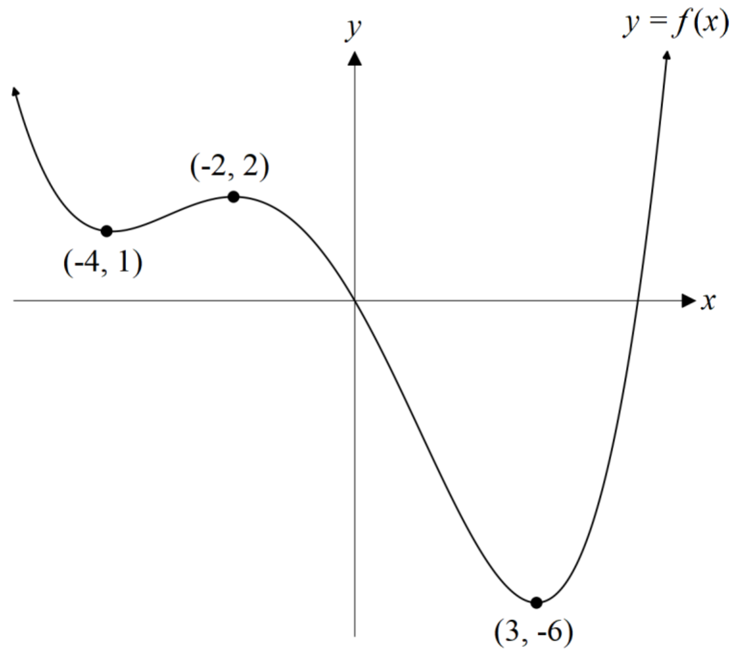
Matty and Charlie are playing a game where they take turns tossing their own coin. Matty's coin is a fair coin, while Charlie's coin has a $\frac{1}{3}$ chance of showing heads. Matty goes first and the first person whose coin toss shows a head wins the game.

(a) By drawing a probability tree or otherwise, find the probability that Charlie wins the game on her first or second toss. **2**

(b) What is the probability that Charlie will win the game? **2**

Question 36 (4 marks)

The graph below shows $y = f(x)$ with all stationary points labelled.



(a) On the same axes above, sketch $y = f'(x)$. **2**

(b) Without finding the equation of $y = f'(x)$, find the area bounded by the curve $y = f'(x)$ and the x-axis. **2**

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End of paper

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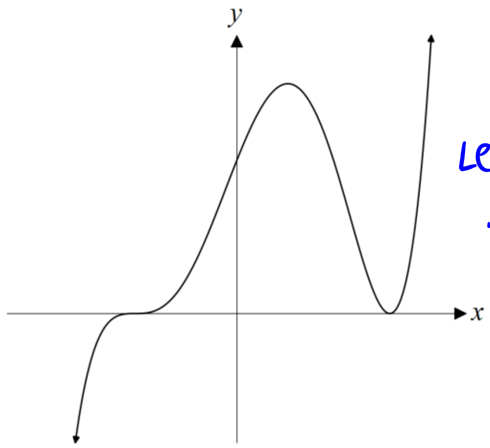
$A = P(1+r)^n$
$= 6000(1.065)^{15}$
$\doteqdot 15431.0\ldots$
$I = A - P$
$= 15431.0\ldots - 6000$
$= 9431.0\ldots$
$\doteqdot 9431$

2. Which of the following could be the graph of $y = (x+3)^2(2-x)^3$?

bounce at -3 wiggle at 2
Leading coeff < 0
$\therefore$ Q4

A.



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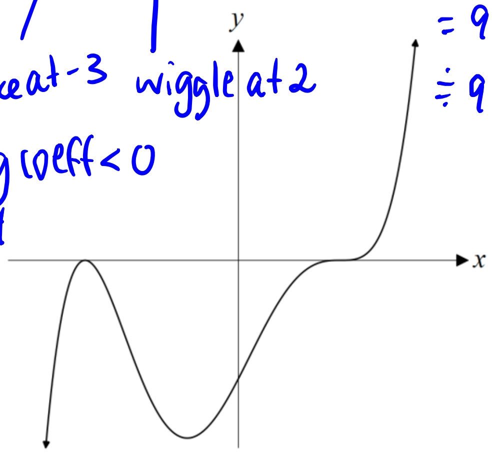

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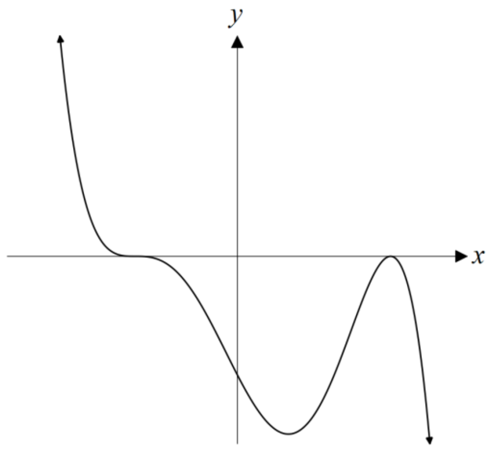
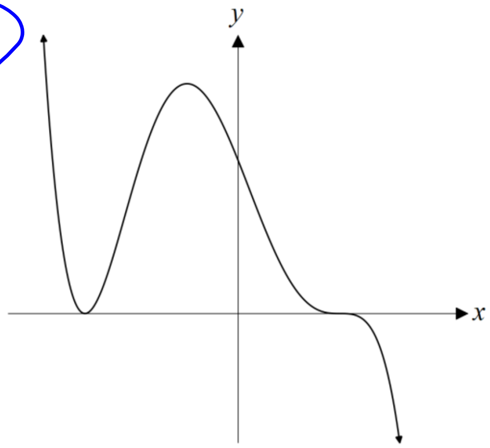


D.



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73	56.05643	55.12993	54.22502	53.34111	52.47764
74	56.63169	55.68446	54.75957	53.85641	52.97438

$PV = MX$
$= 100 \times 53.68593$
$= 5368.593$
$\doteqdot 5368.59$

$n = 6 \times 12$ months
$= 72$

Liv is planning to invest $100 at the end of each month at an interest rate of 0.0085 per month for 6 years. What single amount could Annabelle invest now at the same interest rate, in order to achieve the same future value as Liv in 6 years time?

A. $3 914.42 B. $5 368.59 C. $7 200 D. $13 243.34

4. Which expression shows $\dfrac{45^{2-k}}{6^k}$ as a product of its prime factors in simplified form?

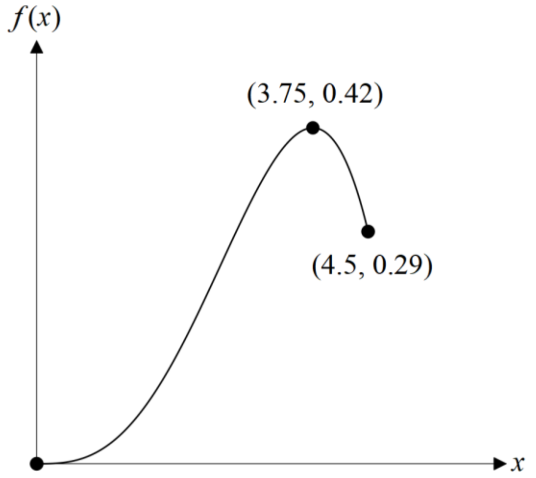
A. $\dfrac{15^{6-4k}}{2^k}$ B. $\dfrac{15^{6-2k}}{2^k}$ C. $\dfrac{3^{4-3k} \times 5^{2-k}}{2^k}$ D. $\dfrac{3^{4-k} \times 5^{2-k}}{2^k}$

$\dfrac{(3^2 \times 5)^{2-k}}{(3 \times 2)^k}$

$= \dfrac{3^{4-2k} \times 5^{2-k}}{3^k \times 2^k}$

$= \dfrac{3^{4-3k} \times 5^{2-k}}{2^k}$

5. The probability distribution function of a continuous random variable is shown below.



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6. Which of the following formulas could be used to calculate the sum of the first n positive **even** numbers?

A. $S_n = n(1 + n)$

B. $S_n = 2(2^n - 1)$

C. $S_n = 2n(n + 1)$

D. $S_n = \frac{n}{2}(6 - 2n)$

2, 4, 6, 8

$S_n = \frac{n}{2}(2a+(n-1)d)$

$= \frac{n}{2}(2(2)+(n-1)\times 2)$

$= \frac{n}{2}(4+2n-2)$

$= \frac{n}{2}(2+2n)$

$= n(1+n)$

7. A random variable is normally distributed with a mean of 0 and a standard deviation of 1. The table below gives the probability that this random variable lies below z for some positive values of z.

z	1.30	1.31	1.32	1.33	1.34	1.35	1.36	1.37	1.38	1.39
Probability	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177

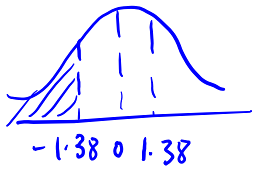
What is $P(z \leq -1.38)$?

A. –0.9162

B. 0.0838

C. 0.4851

D. 0.9162



$= P(z \geq 1.38)$

$= 1 - P(z \leq 1.38)$

$= 1 - 0.9162$

$= 0.0838$

8. The following transformations are applied to the function $f(x) = x^2 - 3$:

- A vertical dilation with scale factor 3.
- Then a horizontal dilation with scale factor 2.
- Then a vertical translation down 1 unit.

Which of the following is the equation of the transformed function $g(x)$?

A. $g(x) = 12x^2 - 4$

B. $g(x) = \frac{3}{4}x^2 - 4$

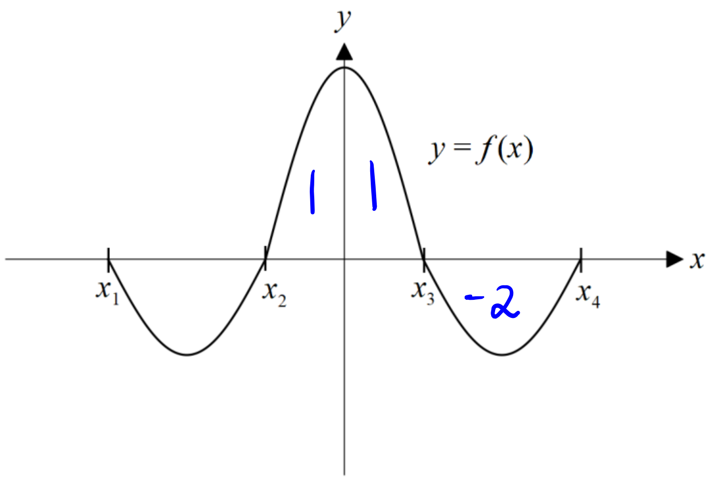
C. $g(x) = 12x^2 - 10$

D. $g(x) = \frac{3}{4}x^2 - 10$

$x^2 - 3 \rightarrow 3x^2 - 9 \rightarrow 3\left(\frac{1}{2}x\right)^2 - 9 \rightarrow \frac{3}{4}x^2 - 10$

$\frac{3}{4}x^2 - 9$

9. The even function $y = f(x)$ is shown below.



It is given that $\int_{x_2}^{x_3} f(x)\,dx = 2$ and $\int_{0}^{x_4} f(x)\,dx = -1$. What is the value of $\int_{x_2}^{x_4} f(x)\,dx$?

A. -1

B. 0

C. 1

D. 2

10. For which values of k is the parabola $y = kx^2 - 2x + 3k + 2$ always positive?

A. $k > 0$

B. $k > \frac{1}{3}$

C. $-1 < k < \frac{1}{3}$

D. $k < -1$ or $k > \frac{1}{3}$

$a > 0$ and $\Delta < 0$

$\therefore k > 0$

$\Delta = b^2 - 4ac$

$= (-2)^2 - 4 \times k(3k+2)$

$= 4 - 12k^2 - 8k$

$= -(12k^2 - 8k + 4$

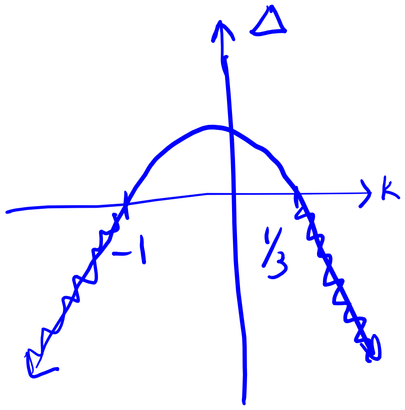
$= -4(3k^2 + 2k - 1)$

$= -4(3k^2 + 3k - k - 1)$

$= -4(3k(k+1) - (k+1))$

$= -4(3k-1)(k+1)$

End of Multiple Choice.



Mathematics Advanced

Section II Answer Booklet

90 marks
Attempt Questions 11 – 36.
Allow about 2 hours and 45 minutes for this section.

Instructions:

- Answer each question in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations
- Additional writing space is available on pages 33 to 41. If you use this space, please start a new sheet of paper for each question, label it with your NESA number and teacher, and clearly note which question you are answering.

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Question 11 (2 marks)

A parabola with vertex (2, –5) has a y-intercept of 11. Find the equation of the parabola. **2**

$y = a(x-h)^2 + k$

$y = a(x-2)^2 - 5$ ✓

when $x = 0$, $y = 11$

$11 = a(0-2)^2 - 5$

$= 4a - 5$

$16 = 4a$

$a = 4$

$\therefore y = 4(x-2)^2 - 5$ ✓ expanded form also OK

Question 12 (2 marks)

If $3 + \dfrac{1}{x} = \sqrt{15}$, find the exact value of x. Express your answer with a rational denominator. **2**

$\dfrac{1}{x} = \sqrt{15} - 3$

$x = \dfrac{1}{\sqrt{15}-3}$ ✓

$x = \dfrac{\sqrt{15}+3}{15-9}$

$= \dfrac{\sqrt{15}+3}{6}$ ✓

Question 13 (3 marks)

A discrete random variable X has probability distribution as shown in the table below, where a and b are real numbers and $E(X) = 2.2$.

x	1	2	3	4	5
$P(X = x)$	0.15	$3a + b$	0	$a - b$	0.05

(a) Find the values of a and b. **2**

$\Sigma p(x) = 1$

$1 = 0.15 + 3a + \cancel{b} + a - \cancel{b} + 0.05$

$= 0.2 + 4a$

$4a = 0.8$

$a = 0.2$ ✓

$\Sigma x p(x) = 2.2$

$2.2 = 0.15 + 2(3 \times 0.2 + b) + 4(0.2 - b) + 5 \times 0.05$

$= 2.4 + 2b - 4b$

$-0.2 = -2b$

$b = 0.1$ ✓

$\therefore a = 0.2$ and $b = 0.1$

(b) Find Var(X). **1**

$Var(X) = E(X^2) - \mu^2$

$= 1^2 \times 0.15 + 2^2(3 \times 0.2 + 0.1) + 4^2(0.2 - 0.1) + 5^2 \times 0.05 - 2.2^2$ ✓

$= 5.8 - 4.84$ Full marks for correct substitution

$= 0.96$

Question 14 (2 marks)

Differentiate $y = \ln\left(\dfrac{x^2}{2x+1}\right)$. **2**

$y = \ln(x^2) - \ln(2x+1)$

$= 2\ln x - \ln(2x+1)$ ✓

$y' = \dfrac{2}{x} - \dfrac{2}{2x+1}$ ✓

Question 15 (2 marks)

Find $\int x^2(1-x^3)^5\,dx$. **2**

$= -\frac{1}{3}\int -3x^2(1-x^3)^5\,dx$ ✓ reverse chain

$= -\dfrac{1}{3}\dfrac{(1-x^3)^6}{6} + C$

$= -\dfrac{(1-x^3)^6}{18} + C$ ✓ integral including +c

Question 16 (2 marks)

Use 3 applications of the trapezoidal rule to estimate $\int_0^{12} \sqrt{x}\,dx$. Give your answer correct to 1 decimal place. **2**

x	0	4	8	12
$f(x)$	0	2	$2\sqrt{2}$	$2\sqrt{3}$

✓

$\int_0^{12} \sqrt{x}\,dx \doteqdot \dfrac{12-0}{2\times 3}\left(0 + 2\sqrt{3} + 2(2 + 2\sqrt{2})\right)$

$\doteqdot 26.24\ldots$

$\doteqdot 26.2$ ✓ including rounding

Question 17 (6 marks)

When Alexia plugs her phone in to charge, the battery percentage displayed on her phone is given by the equation $B = -50e^{-0.6t} + 92$. B is the battery percentage and t is the number of hours the phone has been charging for.

(a) Sketch the graph of B for $t \geq 0$. **3**

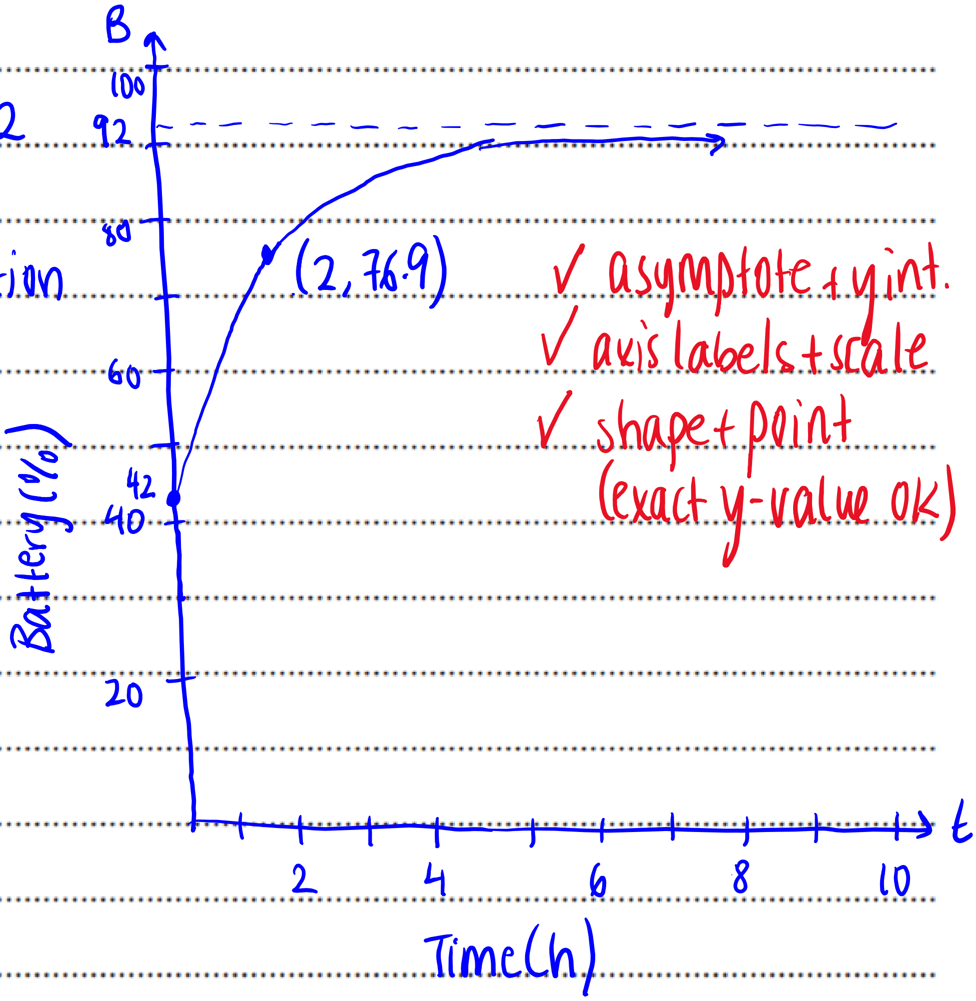
when $t=0$

$B = -50e^{0} + 92$

$= 42$

Double reflection

up 92



✓ asymptote + y int.

✓ axis labels + scale

✓ shape + point

(exact y-value ok)

(b) What is the maximum charge capacity of Alexia's phone battery? **1**

92% ✓

(c) At what rate is Alexia's phone charging after being plugged in for 2 hours? Give your answer rounded to the nearest whole. **2**

$B' = -0.6 \times -50e^{-0.6t}$

$= 30e^{-0.6t}$ ✓

when $t=2$, $B' = 30e^{-0.6\times 2}$

$= 9.0\ldots$

$\doteqdot 9$ ✓

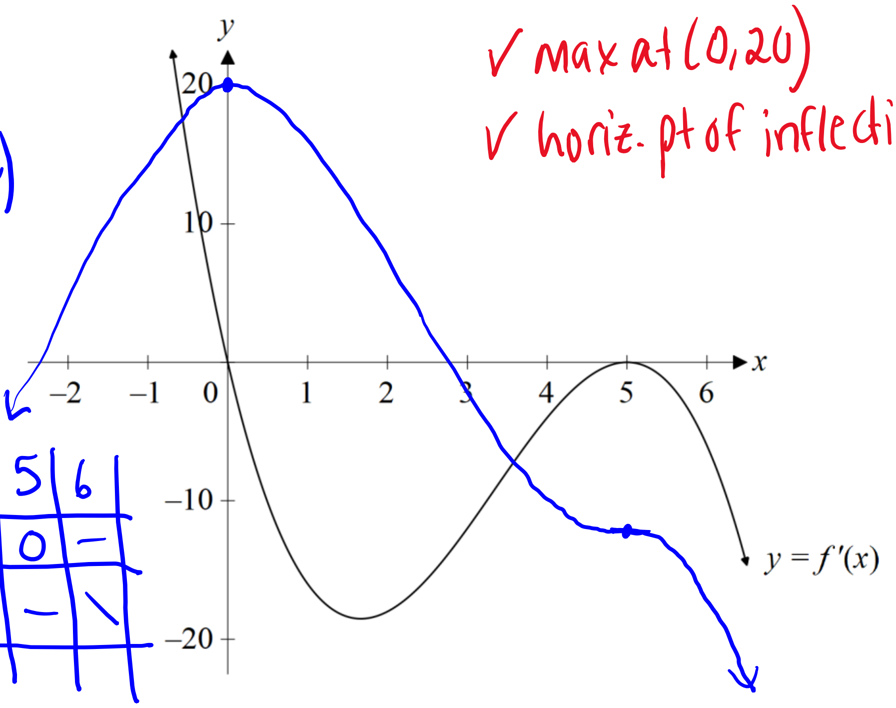
$\therefore$ Charging at 9%/h

Question 18 (2 marks)

The graph of $y = f'(x)$ is shown below. Sketch a graph of $y = f(x)$ on the same axes, given that $f(0) = 20$. **2**

✓ max at (0,20)

✓ horiz. pt of inflection at $x = 5$



x	-1	0	1	5	6
sign $f'(x)$	$+$	0	$-$	0	$-$
slope $f(x)$	/	—	\	—	\

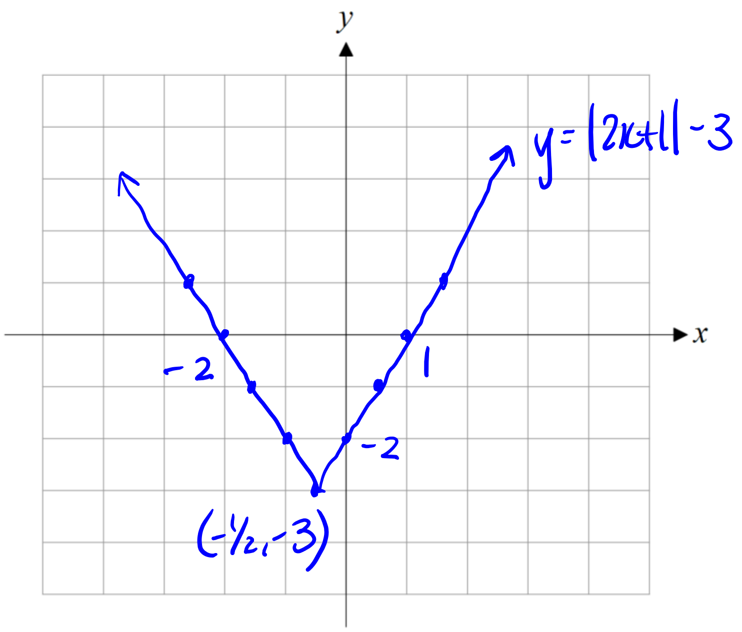
Question 19 (3 marks)

(a) On the axes below, graph the function $y = |2x + 1| - 3$ labelling all important features. **2**

$y = |2(x + \tfrac{1}{2})| - 3$

$\therefore$ Left $\tfrac{1}{2}$, down 3

$m = 2$



✓ intercepts

✓ vertex + shape

(b) Hence, or otherwise, solve $|2x + 1| - 3 \leq 0$. **1**

$-2 \leq x \leq 1$ ✓

Question 20 (2 marks)

Find $\int \sin\left(\frac{x}{2}\right)\, dx$. **2**

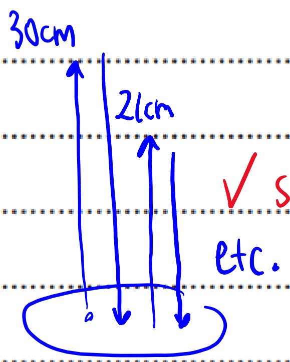
$= 2\int \frac{1}{2}\sin\left(\frac{x}{2}\right) dx$

$= -2\cos\left(\frac{x}{2}\right) + C$

✓ reverse chain ✓ correct integral with +C

Question 21 (3 marks)

Amalie is bouncing on a trampoline. Her first bounce takes her to a height of 30 cm above the trampoline. She then lands on the trampoline and continues to bounce. Each time Amalie bounces, she rises to 70% of her previous height above the trampoline. Find the total distance Amalie will cover eventually. **3**



✓ sequence

Total distance $= 2S_\infty$ ✓

$= 2 \times \frac{a}{1-r}$

$= 2 \times \frac{30}{1-0.7}$ ✓

$= 200$

$\therefore$ Amalie eventually travels 200cm

Question 22 (3 marks)

A function $y = f(x)$ undergoes the following transformations:

- A reflection in the x axis
- Then a horizontal dilation with scale factor $\frac{1}{3}$
- Then a horizontal translation 7 units left

The point $P(x, y)$ lies on $y = f(x)$. The image of P on the transformed function is $P'(-2, 1)$. Find the coordinates of P. **3**

$a = 3,\ b = 7,\ k = -1$ ✓

$x_2 = \frac{1}{a}x - b$ $y_2 = ky + c$

$-2 = \frac{1}{3}x - 7$ $1 = -y$ ✓ logical working

$5 = \frac{1}{3}x$ $y = -1$

$x = 15$

$\therefore P(15, -1)$ ✓

Question 23 (3 marks)

Consider the functions $f(x) = x^2 - 6x$ and $g(x) = \sqrt{x + 5}$.

(a) Find an equation for the composite function $g(f(x))$. **1**

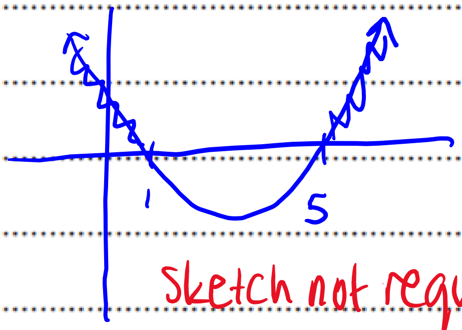
$g(f(x)) = \sqrt{x^2 - 6x + 5}$ ✓

(b) Hence find the domain of $g(f(x))$. **2**

$x^2 - 6x + 5 \geq 0$ ✓

$(x-5)(x-1) \geq 0$

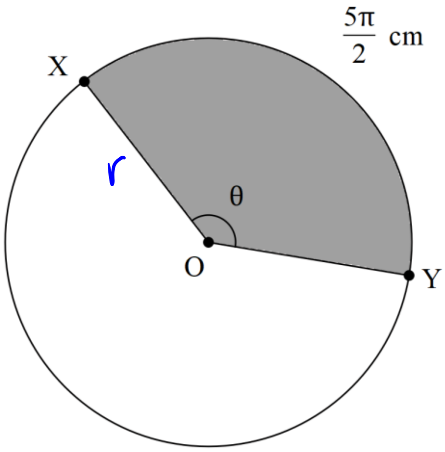
$x \leq 1$ or $x \geq 5$ ✓



Sketch not required

Question 24 (3 marks)

In the diagram below, the length of minor arc XY is $\frac{5\pi}{2}$ cm, θ is the angle subtended by that arc, and the area of shaded sector XOY is $\frac{15\pi}{4}$ cm^2. Find the exact value of θ in radians. **3**



NOT TO SCALE

$L = r\theta$

$\frac{5\pi}{2} = r\theta$

$r = \frac{5\pi}{2\theta}$ (1) ✓

$A = \frac{1}{2}r^2\theta$

$\frac{15\pi}{4} = \frac{1}{2}r^2\theta$ (2)

(1) → (2)

$\frac{15\pi}{4} = \frac{1}{2}\left(\frac{5\pi}{2\theta}\right)^2 \times \theta$ ✓ sim. eq.

$\frac{15\cancel{\pi}}{\cancel{4}} = \frac{1}{2} \times \frac{25\pi^{\cancel{2}}}{\cancel{4}\theta^{\cancel{2}}} \times \cancel{\theta}$

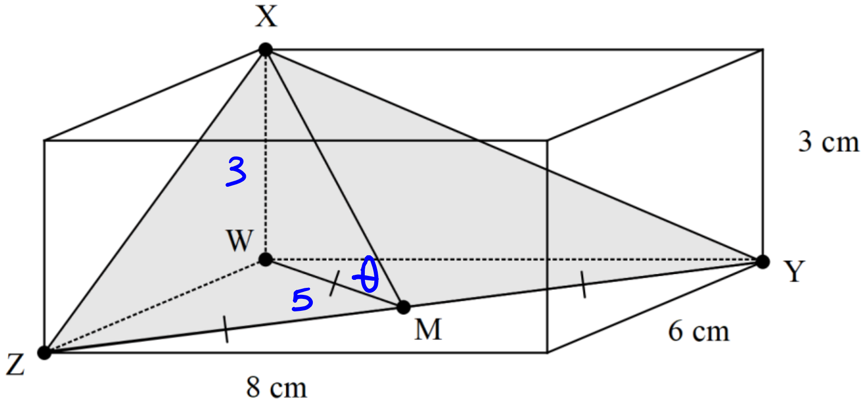
$30 = 25\pi/\theta$

$\theta = 25\pi/30$

$= 5\pi/6$ ✓

Question 25 (2 marks)

The diagram below shows plane *XYZ* inside a rectangular prism. *M* is the midpoint of the base diagonal *YZ*. Find, correct to the nearest degree, the size of angle *WMX*. **2**



$c^2 = a^2 + b^2$

$YZ^2 = 8^2 + 6^2$

$= 100$

$YZ = 10$

$MZ = 5$ ✓

$WM = 5$

$\tan\theta = 3/5$

$\theta = 30.9...^\circ$

$\doteq 31^\circ$

$\therefore \angle WMX \doteq 31^\circ$ ✓ including rounding

Question 26 (2 marks)

Find the gradient of the tangent to the curve $y = \dfrac{\sqrt{x-1}}{x^2}$ at the point where $x = 2$. **2**

$y' = \dfrac{vu' - uv'}{v^2}$ $\quad u = (x-1)^{1/2}$ $\quad v = x^2$

$u' = \frac{1}{2}(x-1)^{-1/2}$ $\quad v' = 2x$

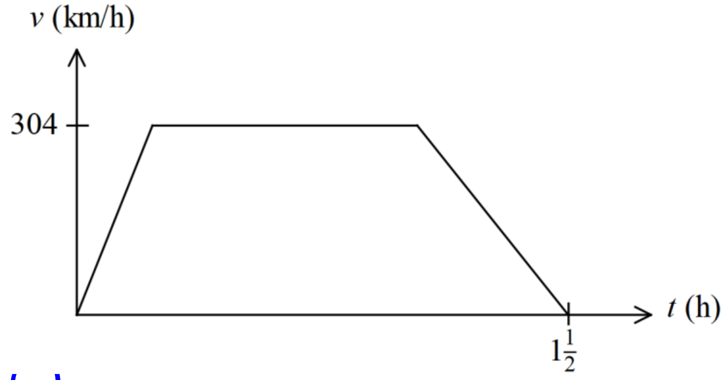
$= \dfrac{\frac{1}{2}x^2(x-1)^{-1/2} - 2x(x-1)^{1/2}}{x^4}$ ✓

when $x = 2$, $y' = \dfrac{\frac{1}{2}(2)^2(2-1)^{-1/2} - 2(2)(2-1)^{1/2}}{2^4}$

$m_T = -1/8$ ✓

Question 27 (2 marks)

A bullet train is travelling from Sendai to Tokyo. The train leaves Sendai and accelerates at a constant rate until it reaches 304 km/h. It continues at this speed for 45 minutes before decelerating at a constant rate and coming to a stop in Tokyo. The total travel time is $1\frac{1}{2}$ hours. By considering the velocity-time graph shown below, find the total distance the train travelled. **2**



$A = \frac{h}{2}(a+b)$

$= \frac{304}{2}(0.75 + 1.5)$ ✓

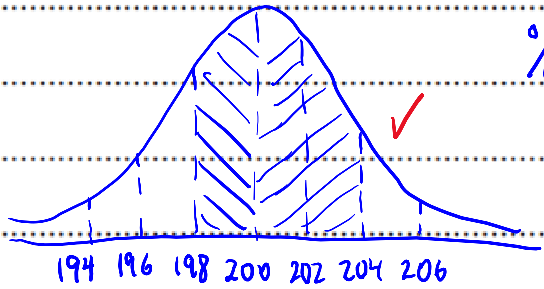
$= 342$

$\therefore$ Train travelled 342 km ✓

Question 28 (5 marks)

Tish runs a business selling packets of stickers. The number of stickers per packet is normally distributed with a mean of 200 and a standard deviation of 2. In order to balance customer satisfaction and profits, Tish only sells packets that contain between 198 and 204 stickers.

(a) If Tish produces 400 packets of stickers per month, how many of these can she sell? **3**



% can sell $= \frac{1}{2} \times 68\% + \frac{1}{2} \times 95\%$

$= 81.5\%$ ✓

Number can sell $= 81.5\% \times 400$

$= 326$ ✓

(b) Given that a packet is unable to be sold, what is the probability that it contains fewer stickers than average. Give your answer to the nearest percent. **2**

$P(\text{too few} \mid \text{can't sell}) = \dfrac{P(\text{too few} \cap \text{can't sell})}{P(\text{can't sell})}$

$= \dfrac{0.5 - 0.5 \times 0.68}{1 - 0.815}$ ✓

$= 0.864\ldots$

$\doteqdot 86\%$ ✓

no marks off for rounding

Question 29 (4 marks)

May has a mango orchard with 20 trees that each produce 400 mangoes per year. For every additional tree that May plants in the orchard, the output of every tree decreases by 10 mangoes per year.

(a) Show that the number of mangoes, M, produced by May's orchard each year is given by $M = -10x^2 + 200x + 8000$ where x is the number of additional trees planted. **2**

trees $= 20 + x$ # mangoes per tree $= 400 - 10x$

$M = (20+x)(400-10x)$ ✓

$= 8000 - 200x + 4000x - 10x^2$

$= -10x^2 + 200x + 8000$ ✓

(b) Hence find the number of additional trees May should plant in order to maximise the number of mangoes grown each year. **2**

$M' = -20x + 200$

Stationary points when $M' = 0$

$0 = -20x + 200$ Full marks for $x = \frac{-b}{2a}$ method

$20x = 200$

$x = 10$ ✓

$M'' = -20$

< 0 for all x $\therefore$ local max at $x = 10$ ✓

$\therefore$ May should plant 10 more trees

Question 30 (5 marks)

Let $f(x) = \cos\left(3x + \dfrac{\pi}{2}\right) - 1$.

(a) Sketch $y = f(x)$ in the domain $0 \leq x \leq \pi$, labelling all important features. **3**

$f(x) = \cos(3(x + \pi/6)) - 1$

$T = 2\pi/3$

$b = \pi/6$, $c = -1$

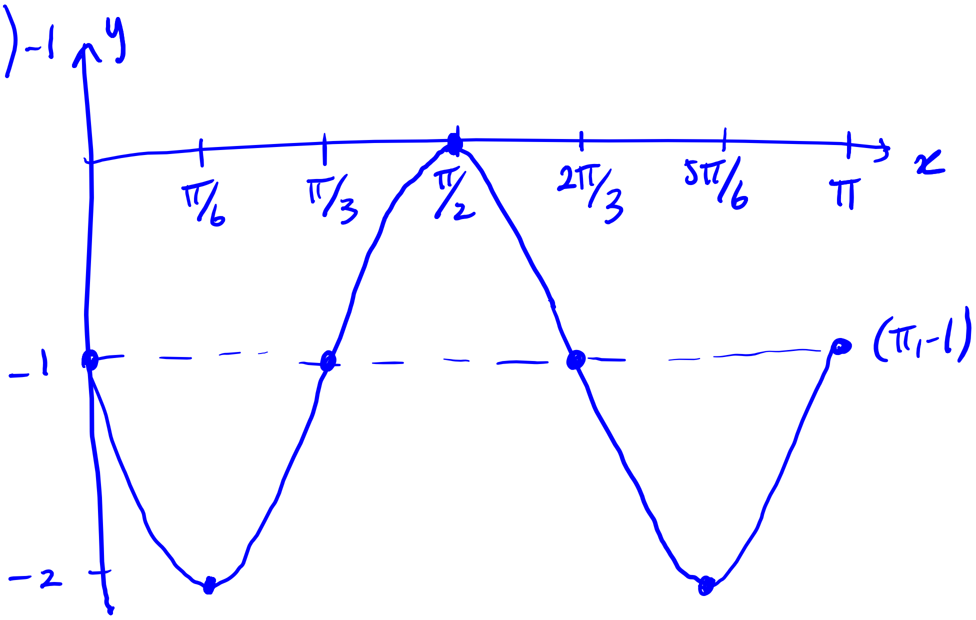
shifted down 1

left $\pi/6$

✓ phase + centre

✓ scale + labels

✓ period + shape



(b) Hence, or otherwise, solve $\cos\left(3x + \dfrac{\pi}{2}\right) = -1$ for $0 \leq x \leq \pi$. **2**

$\cos(3x + \pi/2) - 1 = -1 - 1$ ✓

$= -2$

$\therefore x = \pi/6, 5\pi/6$ ✓ or by solving trig. equation

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Question 31 (8 marks)

At the beginning of January 2023, Hannah decides that she wants to go travelling after finishing Year 12. To save for her trip, she starts investing \$100 at the end of each month into a bank account which earns interest at a rate of 6% p.a. compounded monthly.

The table below shows the future value interest factors for an annuity of \$1 invested at the end of each period.

Periods	Interest rate per period							
n	0.25%	0.5%	1.00%	2.00%	3.00%	4.00%	5.00%	6.00%
6	6.0376	6.0755	6.1520	6.3081	6.4684	6.6330	6.8019	6.9753
7	7.0527	7.1059	7.2135	7.4343	7.6625	7.8983	8.1420	8.3938
8	8.0704	8.1414	8.2857	8.5830	8.8923	9.2142	9.5491	9.8975
9	9.0905	9.1821	9.3685	9.7546	10.1591	10.5828	11.0266	11.4913
10	10.1133	10.2280	10.4622	10.9497	11.4639	12.0061	12.5779	13.1808
11	11.1385	11.2792	11.5668	12.1687	12.8078	13.4864	14.2068	14.9716
12	12.1664	12.3356	12.6825	13.4121	14.1920	15.0258	15.9171	16.8699

(a) Use the future value table above to find Hannah's account balance at the end of December 2023. **1**

$FV = MX$ $\quad n = 1y = 12m \quad r = 6\%\text{ p.a.} = 0.5\%\text{ p.m.}$

$= 100 \times 12.3356$

$= 1233.56$

$\therefore$ Hannah has \$1233.56 ✓

(b) At the beginning of January 2024, Hannah starts working at a new job and decides to increase her monthly investments to \$200. If Hannah continues this for the year, show that her account balance at the end of December 2024 is \$3776.76. **2**

$FV = FV_{LUMP} + FV_{INSTAL.}$ $\quad n = 12m \quad r = 0.5\%\text{ p.m.}$

$= P(1+r)^n + MX$

$= 1233.56(1+0.5\%)^{12} + 200 \times 12.3356$ ✓

$= 3776.763\ldots$

$\doteqdot 3776.76$ ✓

$\therefore$ Hannah has \$3776.76

Question 31 continues on page 24

Question 31 (continued)

(c) At the beginning of January 2025, Hannah starts her trip and stops contributing to the account. Instead, she withdraws \$400 from the account **at the start** of each month to cover expenses. The account continues to earn interest at 6% p.a. compounded monthly. Show that Hannah's account balance, A_n, at the end of the nth month of her trip is given by:

$$A_n = 3776.76(1.005)^n - 80400(1.005^n - 1)$$

3

$A_1 = (3776.76 - 400)(1.005)$ ← not required

$= 3776.76(1.005) - 400(1.005)$

$A_2 = (A_1 - 400)(1.005)$

$= 3776.76(1.005)^2 - 400(1.005)^2 - 400(1.005)$

$A_3 = (A_2 - 400)(1.005)$

$= 3776.76(1.005)^3 - 400(1.005)^3 - 400(1.005)^2 - 400(1.005)$ ✓

$\vdots$

$A_n = 3776.76(1.005)^n - 400(1.005)^n - 400(1.005)^{n-1} - \dots - 400(1.005)$

$= 3776.76(1.005)^n - 400(1.005 + \dots + 1.005^{n-1} + 1.005^n)$ ✓

$= 3776.76(1.005)^n - 400 \times \dfrac{1.005(1.0005^n - 1)}{1.005 - 1}$ ✓

$A_n = 3776.76(1.005)^n - 80400(1.005^n - 1)$

GP with

$a = 1.005$

$r = 1.005$

$n = n$

$S_n = \dfrac{a(r^n - 1)}{r - 1}$

Question 31 continues on page 25

Question 31 (continued)

(d) How many $400 withdrawals can Hannah make from her travel fund? **2**

Fund runs out when $A_n = 0$

$0 = 3776.76(1.005)^n - 80400(1.005)^n + 80400$

$= -76623.24(1.005)^n + 80400$ ✓

$76623.24(1.005)^n = 80400$

$(1.005)^n = 1.049...$

$n \ln 1.005 = \ln 1.049...$

$n = 9.64...$

$\therefore$ Hannah can make 9 withdrawals ✓ including rounding down

End of Question 31

Question 32 (4 marks)

The time in minutes taken for Claudia to score her first point in a tennis match can be represented by a continuous random variable X with probability density function given by:

$$f(x) = \begin{cases} ke^x & \text{for } 0 \le x \le 4 \\ ke & \text{for } 4 < x \le 10 \\ 0 & \text{for all other } x \end{cases}$$

(a) Find the exact value of the constant k. **2**

$1 = \int_0^4 ke^x\,dx + \int_4^{10} ke\,dx$

$= [ke^x]_0^4 + [kex]_4^{10}$ ✓

$= ke^4 - ke^0 + 10ke - 4ke$

$= ke^4 - k + 6ke$

$= k(e^4 + 6e - 1)$

$k = \dfrac{1}{e^4+6e-1}$ ✓

(b) The median time taken for Claudia to score her first point is less than 4 minutes. Find this median time, giving your answer correct to the nearest second. **2**

$0.5 = \int_0^x \dfrac{1}{e^4+6e-1} e^t\,dt$ ✓

$= \dfrac{1}{e^4+6e-1}[e^t]_0^x$

$34.9... = e^x - e^0$

$= e^x - 1$

$e^x = 35.9...$

$\ln e^x = \ln 35.9...$

$x = 3°34'56.06''$

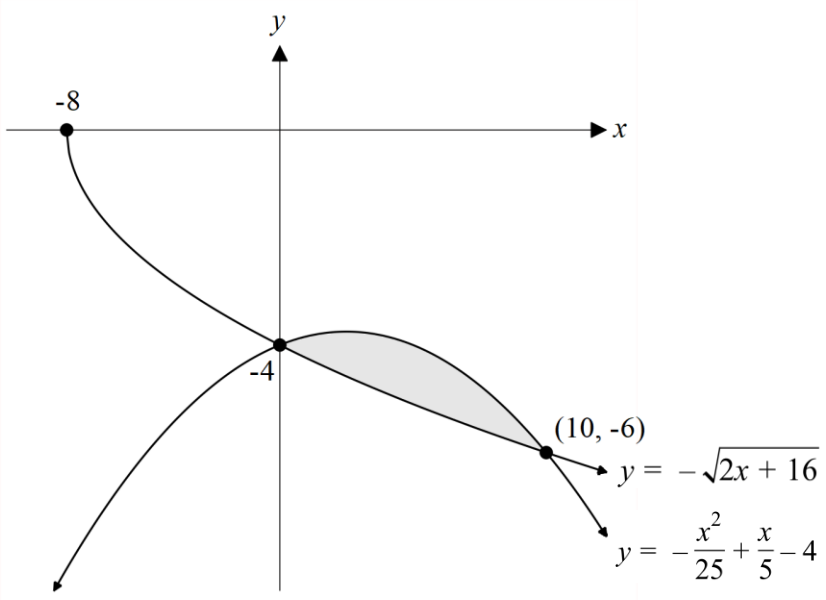
$\doteqdot 3°35'$

$\therefore$ Claudia's median time to score a point is 3 min 35 sec ✓

no marks off for incorrect rounding

Question 33 (4 marks)

The diagram below shows the graphs of $y = -\sqrt{2x+16}$ and $y = -\dfrac{x^2}{25} + \dfrac{x}{5} - 4$. Find the area of the shaded region. **4**



$$A = \int_0^{10} -\frac{x^2}{25} + \frac{x}{5} - 4 - -(2x+16)^{1/2}\,dx$$

$$= \int_0^{10} -\frac{x^2}{25} + \frac{x}{5} - 4\,dx + \frac{1}{2}\int_0^{10} 2(2x+16)^{1/2}\,dx$$

$$= \left[\frac{-x^3}{75} + \frac{x^2}{10} - 4x\right]_0^{10} + \frac{1}{\cancel{2}}\left[\frac{\cancel{2}(2x+16)^{3/2}}{3}\right]_0^{10}$$

$$= \left(\frac{-10^3}{75} + \frac{10^2}{10} - 4(10) - 0\right) + \left(\frac{(2(10)+16)^{3/2}}{3} - \frac{(2(0)+16)^{3/2}}{3}\right)$$

$$= \frac{-130}{3} + \frac{152}{3}$$

$$= \frac{22}{3} \text{ units}^2$$

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Question 34 (8 marks)

Let $f(x) = 10e^{-x}(1-x)$.

(a) Find the exact coordinates of any stationary points on $y = f(x)$ and determine their nature. **3**

$f'(x) = vu' + uv'$ $\quad u = 10e^{-x} \quad v = 1-x$

$= -10e^{-x}(1-x) - 10e^{-x}$ $\quad u' = -10e^{-x} \quad v' = -1$

$= -10e^{-x}(1-x+1)$

$= -10e^{-x}(2-x)$ ✓

Stationary points when $f'(x) = 0$

$0 = -10e^{-x}(2-x)$

$-10e^{-x} = 0$ or $2-x=0$

no solns. $\quad x = 2$

$f''(x) = vu' + uv'$ $\quad u = -10e^{-x} \quad v = 2-x$

$= 10e^{-x}(2-x) + 10e^{-x}$ $\quad u' = 10e^{-x} \quad v' = -1$

$= 10e^{-x}(2-x+1)$

$= 10e^{-x}(3-x)$

$f''(2) = 10e^{-2}(3-2)$ $\quad f(2) = 10e^{-2}(1-2)$

$= 1.3\ldots$ $\quad = -10e^{-2}$

$> 0 \therefore$ local min ✓

$\therefore (2, -10e^{-2})$ is a local minimum

✓

Question 34 continues on page 30

Question 34 (continued)

(b) Find the exact coordinates of any points of inflection on $y = f(x)$. **2**

Possible points of inflection when $f''(x) = 0$

$0 = 10e^{-x}(3-x)$

$10e^{-x} = 0$ or $3 - x = 0$

no solns. $x = 3$

x	2.5	3	3.5
$f''(x)$	0.4...	0	-0.4..
conc.	∪	*	∩

$f(3) = 10e^{-3}(1-3)$

$= -20e^{-3}$

Concavity change ∴ point of inflection ✓

∴ $(3, -20e^{-3})$ is a point of inflection

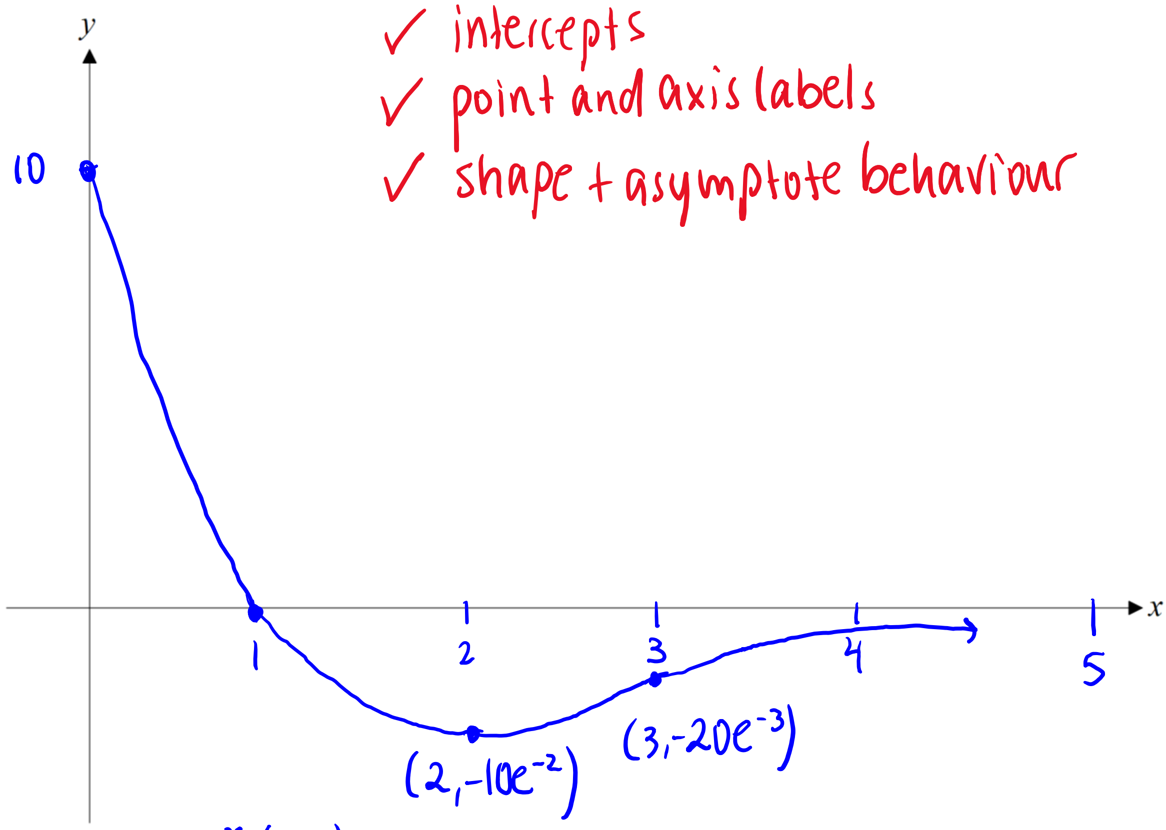
✓

(c) Hence sketch the graph of $y = f(x)$ for $x \geq 0$, showing all important features. **3**

✓ intercepts

✓ point and axis labels

✓ shape + asymptote behaviour



When $y = 0$, $0 = 10e^{-x}(1-x)$

$10e^{-x} = 0$ or $1 - x = 0$

no solns. $x = 1$

As $x \to 0$, $f(x) \to \infty$

As $x \to \infty$, $f(x) \to 0^-$

End of Question 34

Question 35 (4 marks)

Matty and Charlie are playing a game where they take turns tossing their own coin. Matty's coin is a fair coin, while Charlie's coin has a $\frac{1}{3}$ chance of showing heads. Matty goes first and the first person whose coin toss shows a head wins the game.

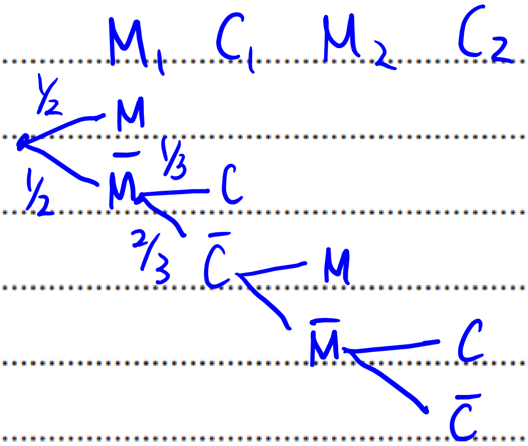
(a) By drawing a probability tree or otherwise, find the probability that Charlie wins the game on her first or second toss. **2**

$P(\text{Charlie W 1st or 2nd}) = P(\bar{M}C) + P(\bar{M}\bar{C}\bar{M}C)$

$= \frac{1}{2} \times \frac{1}{3} + \frac{1}{2} \times \frac{2}{3} \times \frac{1}{2} \times \frac{1}{3}$ ✓

$= \frac{2}{9}$ ✓

First mark can be for tree diagram if drawn and labelled correctly



(b) What is the probability that Charlie will win the game? **2**

$P(\text{C W eventually}) = P(\bar{M}C) + P(\bar{M}\bar{C}\bar{M}C) + P(\bar{M}\bar{C}\bar{M}\bar{C}\bar{M}C) + \dots$

$= \frac{1}{2} \times \frac{1}{3} + \frac{1}{2} \times \underbrace{\frac{2}{3} \times \frac{1}{2}} \times \frac{1}{3} + \frac{1}{2} \times \frac{2}{3} \times \frac{1}{2} \times \underbrace{\frac{2}{3} \times \frac{1}{2}} \times \frac{1}{3} + \dots$ ✓

Infinite sum of a GP with $a = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$, $r = \frac{2}{3} \times \frac{1}{2} = \frac{1}{3}$ ✓

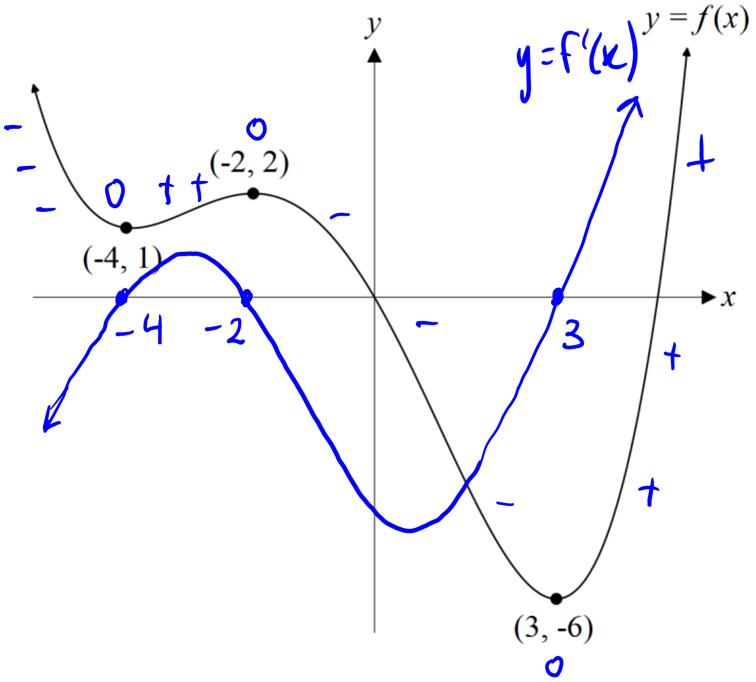
$= \frac{a}{1-r}$

$= \frac{1}{6} \div \left(1 - \frac{1}{3}\right)$

$= \frac{1}{4}$ ✓

Question 36 (4 marks)

The graph below shows $y = f(x)$ with all stationary points labelled.



✓ x-intercepts
✓ shape

(a) On the same axes above, sketch $y = f'(x)$. **2**

(b) Without finding the equation of $y = f'(x)$, find the area bounded by the curve $y = f'(x)$ and the x-axis. **2**

$\text{Area} = \int_{-4}^{-2} f'(x)\,dx + \left|\int_{-2}^{3} f'(x)\,dx\right|$

$= [f(x)]_{-4}^{-2} + \left|[f(x)]_{-2}^{3}\right|$ ✓

$= f(-2) - f(-4) + |f(3) - f(-2)|$

$= 2 - 1 + |-6 - 2|$

$= 1 + |-8|$

$= 9 \text{ units}^2$ ✓ no marks off for no units

End of paper

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Mathematics Advanced Trial Examination 2024

Section I Answer Sheet

1. A B C D

2. A B C D

3. A B C D

4. A B C D

5. A B C D

6. A B C D

7. A B C D

8. A B C D

9. A B C D

10. A B C D

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